Equivalence between new and old forms for vorticity

$$\mathbf{U} = \mathbf{X}\times\mathbf{P} / |\mathbf{X}\times\mathbf{P}| = (0, -r, q) / \sqrt{(qq+rr)} = (0, -\sin\phi, \sin\lambda \cos\phi) / \cos\delta$$
$$\mathbf{V} = \mathbf{Y}\times\mathbf{P} / |\mathbf{Y}\times\mathbf{P}| = (r, 0, -p) / \sqrt{(rr+pp)} = (\sin\phi, 0, -\cos\lambda \cos\phi) / \cos\varepsilon$$
$$\mathbf{W} = \mathbf{Z}\times\mathbf{P} / |\mathbf{Z}\times\mathbf{P}| = (-q, p, 0) / \sqrt{(pp+qq)} = (-\sin\lambda, \cos\lambda, 0)$$
$$\mathbf{N} = \mathbf{P}\times\mathbf{W} = (-rp, -qr, pp+qq) / \sqrt{(pp+qq)}$$

$$\partial\lambda/\partial\mu = -rp / (pp+qq) = -\cos\lambda \tan\varphi$$
$$\partial\lambda/\partial\nu = -qr / (pp+qq) = -\sin\lambda \tan\varphi$$
$$\partial\varphi/\partial\mu = q / \sqrt{(pp+qq)} = \sin\lambda$$
$$\partial\varphi/\partial\nu = -p / \sqrt{(pp+qq)} = -\cos\lambda$$

$$a = u \cos\delta = (w\, \mathbf{U}\bullet\mathbf{W} + n\, \mathbf{U}\bullet\mathbf{N}) \cos\delta = (-w \cos\lambda \sin\phi + n \sin\lambda)$$
$$b = v \cos\varepsilon = (w\, \mathbf{V}\bullet\mathbf{W} + n\, \mathbf{V}\bullet\mathbf{N}) \cos\varepsilon = (-w \sin\lambda \sin\phi - n \cos\lambda)$$
$$c = w \cos\phi$$

$$\partial(rb-qc)/\partial\mu = \partial[\sin\phi\, (-w \sin\lambda \sin\phi - n \cos\lambda) - \sin\lambda \cos\phi\, w \cos\phi]/\partial\mu =$$
$$= \partial(-w \sin\lambda - n \cos\lambda \sin\phi)/\partial\mu$$
$$\partial(pc-ra)/\partial\nu = \partial(w \cos\lambda - n \sin\lambda \sin\phi)/\partial\nu$$
$$\partial(qa-pb)/\partial\lambda = \partial(n \cos\phi)/\partial\lambda$$

$$\zeta = [\partial(rb-qc)/\partial\mu + \partial(pc-ra)/\partial\nu + \partial(qa-pb)/\partial\lambda] / R =$$

$$= [\partial\lambda/\partial\mu\, \partial(-w \sin\lambda - n \cos\lambda \sin\phi)/\partial\lambda + \partial\phi/\partial\mu\, \partial(-w \sin\lambda - n \cos\lambda \sin\phi)/\partial\phi +$$
$$+ \partial\lambda/\partial\nu\, \partial(w \cos\lambda - n \sin\lambda \sin\phi)/\partial\lambda + \partial\phi/\partial\nu\, \partial(w \cos\lambda - n \sin\lambda \sin\phi)/\partial\phi + \partial(n \cos\phi)/\partial\lambda] / R$$

$$= [-\cos\lambda \tan\phi\, (-w \cos\lambda - \sin\lambda\, \partial w/\partial\lambda + n \sin\lambda \sin\phi - \cos\lambda \sin\phi\, \partial n/\partial\lambda) +$$
$$+ \sin\lambda\, (-\sin\lambda\, \partial w/\partial\phi - n \cos\lambda \cos\phi - \cos\lambda \sin\phi\, \partial n/\partial\phi) -$$
$$- \sin\lambda \tan\phi\, (-w \sin\lambda + \cos\lambda\, \partial w/\partial\lambda - n \cos\lambda \sin\phi - \sin\lambda \sin\phi\, \partial n/\partial\lambda) -$$
$$- \cos\lambda\, (\cos\lambda\, \partial w/\partial\lambda - n \sin\lambda \cos\phi - \sin\lambda \sin\phi\, \partial n/\partial\phi) + \cos\phi\, \partial n/\partial\lambda] / R =$$

$$= [-\cos\lambda \tan\phi\, (-w \cos\lambda - \cos\lambda \sin\phi\, \partial n/\partial\lambda) + \sin\lambda\, (-\sin\lambda\, \partial w/\partial\phi) -$$
$$- \sin\lambda \tan\phi\, (-w \sin\lambda - \sin\lambda \sin\phi\, \partial n/\partial\lambda) - \cos\lambda\, (\cos\lambda\, \partial w/\partial\lambda) + \cos\phi\, \partial n/\partial\lambda] / R =$$

$$= [w \tan\phi + \tan\phi \sin\phi\, \partial n/\partial\lambda - \partial w/\partial\phi + \cos\phi\, \partial n/\partial\lambda] / R = [\partial n/\partial\lambda - \partial(w \cos\phi)/\partial\phi] / R \cos\phi$$