Advection at a randomly positioned spherical edge

A random edge resides along a meridian between the poles $\mathbf{I}$ and $-\mathbf{I}$.

$$\begin{aligned}
\mathbf{I} &= (i,j,k) = \text{coordinates of "north pole"} && (1)\\
\xi &= \text{longitudinal angular rotation coordinate around } \mathbf{I} \text{ axis} && (2)\\
\eta &= \text{latitudinal angular rotation coordinates from } -\mathbf{I} \text{ to } \mathbf{I} && (3)\\
\sin\eta &= \mathbf{I}\cdot\mathbf{P} = ip + jq + kr && (4)\\
\cos\eta &= |\mathbf{I}\times\mathbf{P}| = \sqrt{[(jr-kq)^2 + (kp-ir)^2 + (iq-jp)^2]} && (5)\\
\mathbf{F} &= \mathbf{I}\times\mathbf{P}/|\mathbf{I}\times\mathbf{P}| = (jr-kq, kp-ir, iq-jp)/\cos\eta = \text{normal to edge} && (6)\\
\mathbf{E} &= \mathbf{P}\times\mathbf{F} = (i - p\sin\eta, j - qsin\eta, k - r\sin\eta)/\cos\eta = \text{parallel to edge} && (7)\\
f &= \mathbf{F}\cdot\mathbf{S} = w\mathbf{F}\cdot\mathbf{W} + n\mathbf{F}\cdot\mathbf{N} = \text{velocity component normal to edge} && (8)\\
e &= \mathbf{E}\cdot\mathbf{S} = w\mathbf{E}\cdot\mathbf{W} + n\mathbf{F}\cdot\mathbf{N} = \text{velocity component parallel to edge} && (9)\\
\mathbf{F}\cdot\mathbf{W} &= (kp^2 + kq^2 - ipr - jqr)/\cos\eta\cos\phi && (10)\\
\mathbf{E}\cdot\mathbf{W} &= (jp - iq)/\cos\eta\cos\phi = (j\cos\lambda - i\sin\lambda)/\cos\eta && (11)\\
\mathbf{F}\cdot\mathbf{N} &= (iq - jp)/\cos\eta\cos\phi = (j\cos\lambda - i\sin\lambda)/\cos\eta = -\mathbf{E}\cdot\mathbf{W} && (12)\\
\mathbf{F}\cdot\mathbf{W} &= (k - r\sin\eta)/\cos\eta\cos\phi = \mathbf{F}\cdot\mathbf{W} && (13)\\
\partial\lambda/\partial\xi &= \mathbf{W}\cdot\mathbf{F}\cos\eta/\cos\phi && (14)\\
\partial\lambda/\partial\eta &= \mathbf{W}\cdot\mathbf{E}/\cos\phi && (15)\\
\partial\phi/\partial\xi &= \mathbf{N}\cdot\mathbf{F}\cos\eta && (16)\\
\partial\phi/\partial\eta &= \mathbf{N}\cdot\mathbf{E} && (17)
\end{aligned}$$

$$(\mathbf{F}\cdot\mathbf{W})^2 + (\mathbf{E}\cdot\mathbf{W})^2 = (\mathbf{F}\cdot\mathbf{N})^2 + (\mathbf{E}\cdot\mathbf{N})^2 = (\mathbf{F}\cdot\mathbf{W})^2 + (\mathbf{F}\cdot\mathbf{N})^2 = (\mathbf{E}\cdot\mathbf{W})^2 + (\mathbf{E}\cdot\mathbf{N})^2 = 1 \quad (18)$$

$$\begin{aligned}
&\frac{\partial\mathbf{A}}{\partial t} + \left(f\frac{\partial\mathbf{A}}{\partial\xi} + e\cos\eta\frac{\partial\mathbf{A}}{\partial\eta}\right)/R\cos\eta =\\
&= \frac{\partial\mathbf{A}}{\partial t} + \left[f\left(\frac{\partial\lambda}{\partial\xi}\frac{\partial\mathbf{A}}{\partial\lambda} + \frac{\partial\phi}{\partial\xi}\frac{\partial\mathbf{A}}{\partial\phi}\right) + e\cos\eta\left(\frac{\partial\lambda}{\partial\eta}\frac{\partial\mathbf{A}}{\partial\lambda} + \frac{\partial\phi}{\partial\eta}\frac{\partial\mathbf{A}}{\partial\phi}\right)\right]/R\cos\eta =\\
&= \frac{\partial\mathbf{A}}{\partial t} + \left[(w\mathbf{F}\cdot\mathbf{W} + n\mathbf{F}\cdot\mathbf{N})\left(\mathbf{W}\cdot\mathbf{F}\cos\eta\frac{\partial\mathbf{A}}{\partial\lambda}/\cos\phi + \mathbf{N}\cdot\mathbf{F}\cos\eta\frac{\partial\mathbf{A}}{\partial\phi}\right)\right.\\
&\qquad \left. + (w\mathbf{E}\cdot\mathbf{W} + n\mathbf{E}\cdot\mathbf{N})\cos\eta\left(\mathbf{W}\cdot\mathbf{E}\frac{\partial\mathbf{A}}{\partial\lambda}/\cos\phi + \mathbf{N}\cdot\mathbf{E}\frac{\partial\mathbf{A}}{\partial\phi}\right)\right]/R\cos\eta =\\
&= \frac{\partial\mathbf{A}}{\partial t} + \left\{w\frac{\partial\mathbf{A}}{\partial\lambda}[(\mathbf{F}\cdot\mathbf{W})^2 + (\mathbf{E}\cdot\mathbf{W})^2]/\cos\phi + w\frac{\partial\mathbf{A}}{\partial\phi}[(\mathbf{F}\cdot\mathbf{W})(\mathbf{N}\cdot\mathbf{F}) + (\mathbf{E}\cdot\mathbf{W})(\mathbf{N}\cdot\mathbf{E})]\right.\\
&\qquad \left. + n\frac{\partial\mathbf{A}}{\partial\lambda}[(\mathbf{F}\cdot\mathbf{N})(\mathbf{W}\cdot\mathbf{F}) + (\mathbf{F}\cdot\mathbf{N})(\mathbf{W}\cdot\mathbf{E})]/\cos\phi + n\frac{\partial\mathbf{A}}{\partial\phi}[(\mathbf{F}\cdot\mathbf{E})^2 + (\mathbf{E}\cdot\mathbf{N})^2]\right\}/R =\\
&= \frac{\partial\mathbf{A}}{\partial t} + \left(w\frac{\partial\mathbf{A}}{\partial\lambda}/\cos\phi + n\frac{\partial\mathbf{A}}{\partial\phi}\right)/R = \frac{\partial\mathbf{A}}{\partial t} + \left(w\frac{\partial\mathbf{A}}{\partial\lambda} + n\cos\phi\frac{\partial\mathbf{A}}{\partial\phi}\right)/R\cos\phi
\end{aligned} \quad (19)$$