Equivalence between new and old forms for $\nabla^2 h$

$$\nabla^2 h = [\partial^2 h/\partial\lambda^2 + \cos\phi\ \partial(\cos\phi\ \partial h/\partial\phi)/\partial\phi] / R^2 \cos^2\phi =$$
$$= [\partial^2 h/\partial\lambda^2 + \cos\phi\ (\cos\phi\ \partial^2 h/\partial\phi^2 - \sin\phi\ \partial h/\partial\phi)] / R^2 \cos^2\phi =$$
$$= [\partial^2 h/\partial\lambda^2 / \cos^2\phi + \partial^2 h/\partial\phi^2 - \sin\phi\ \partial h/\partial\phi / \cos\phi] / R^2 = X$$

$$\partial\lambda/\partial\delta = \mathbf{W}\bullet\mathbf{P}\times\mathbf{U} / \cos\phi = -\sin\lambda / \cos\delta \cos\phi$$
$$\partial\lambda/\partial\varepsilon = \mathbf{W}\bullet\mathbf{P}\times\mathbf{V} / \cos\phi = \cos\lambda / \cos\varepsilon \cos\phi$$
$$\partial\varphi/\partial\delta = N\bullet P\times U = -\cos\lambda \sin\phi / \cos\delta$$
$$\partial\varphi/\partial\varepsilon = N\bullet P\times V = -\sin\lambda \sin\phi / \cos\varepsilon$$

$$\nabla^2 h = \nabla\bullet(\nabla h) =$$
$$= [\cos\delta\ \partial(\cos\delta\ \partial h/\partial\delta)/\partial\delta + \cos\varepsilon\ \partial(\cos\varepsilon\ \partial h/\partial\varepsilon)/\partial\varepsilon + \cos\phi\ \partial(\cos\phi\ \partial h/\partial\phi)/\partial\phi] / R^2 =$$

$$= \{\cos\delta\ \partial[\cos\delta\ (\partial\lambda/\partial\delta\ \partial h/\partial\lambda + \partial\phi/\partial\delta\ \partial h/\partial\phi)]/\partial\delta +$$
$$+ \cos\varepsilon\ \partial(\cos\varepsilon\ (\partial\lambda/\partial\varepsilon\ \partial h/\partial\lambda + \partial\phi/\partial\varepsilon\ \partial h/\partial\phi)]/\partial\varepsilon + \cos\phi\ \partial(\cos\phi\ \partial h/\partial\phi)/\partial\phi\} / R^2 =$$

$$= \{\cos\delta\ \partial[-\sin\lambda\ \partial h/\partial\lambda / \cos\phi - \cos\lambda \sin\phi\ \partial h/\partial\phi]/\partial\delta +$$
$$+ \cos\varepsilon\ \partial(\cos\lambda\ \partial h/\partial\lambda / \cos\phi - \sin\lambda \sin\phi\ \partial h/\partial\phi]/\partial\varepsilon + \cos\phi\ \partial(\cos\phi\ \partial h/\partial\phi)/\partial\phi\} / R^2 =$$

$$= \{\cos\delta\ [\partial\lambda/\partial\delta\ \partial(-\sin\lambda\ \partial h/\partial\lambda / \cos\phi - \cos\lambda \sin\phi\ \partial h/\partial\phi)/\partial\lambda +$$
$$+ \partial\phi/\partial\delta\ \partial(-\sin\lambda\ \partial h/\partial\lambda / \cos\phi - \cos\lambda \sin\phi\ \partial h/\partial\phi)/\partial\phi] +$$
$$+ \cos\varepsilon\ [\partial\lambda/\partial\varepsilon\ \partial(\cos\lambda\ \partial h/\partial\lambda / \cos\phi - \sin\lambda \sin\phi\ \partial h/\partial\phi)/\partial\lambda +$$
$$+ \partial\phi/\partial\varepsilon\ \partial(\cos\lambda\ \partial h/\partial\lambda / \cos\phi - \sin\lambda \sin\phi\ \partial h/\partial\phi)/\partial\phi + \cos\phi\ \partial(\cos\phi\ \partial h/\partial\phi)/\partial\phi\} / R^2 =$$

$$= \{\sin\lambda\ \partial(\sin\lambda\ \partial h/\partial\lambda / \cos\phi + \cos\lambda \sin\phi\ \partial h/\partial\phi)/\partial\lambda / \cos\phi +$$
$$+ \cos\lambda \sin\phi\ \partial(\sin\lambda\ \partial h/\partial\lambda / \cos\phi + \cos\lambda \sin\phi\ \partial h/\partial\phi)/\partial\phi +$$
$$+ \cos\lambda\ \partial(\cos\lambda\ \partial h/\partial\lambda / \cos\phi - \sin\lambda \sin\phi\ \partial h/\partial\phi)/\partial\lambda / \cos\phi +$$
$$- \sin\lambda \sin\phi\ \partial(\cos\lambda\ \partial h/\partial\lambda / \cos\phi - \sin\lambda \sin\phi\ \partial h/\partial\phi)/\partial\phi + \cos\phi\ \partial(\cos\phi\ \partial h/\partial\phi)/\partial\phi\} / R^2 =$$

$$= \{\sin^2\lambda\ \partial^2 h/\partial\lambda^2 / \cos^2\phi + \sin\lambda \cos\lambda\ \partial h/\partial\lambda / \cos^2\phi +$$
$$+ \sin\lambda \cos\lambda \sin\phi\ \partial^2 h/\partial\phi\partial\lambda / \cos\phi - \sin^2\lambda \sin\phi\ \partial h/\partial\phi / \cos\phi +$$
$$+ \cos\lambda \sin\phi \sin\lambda\ \partial^2 h/\partial\lambda\partial\phi / \cos\phi + \cos\lambda \sin^2\phi \sin\lambda\ \partial h/\partial\lambda / \cos^2\phi +$$
$$+ \cos^2\lambda \sin^2\phi\ \partial^2 h/\partial\phi^2 + \cos^2\lambda \sin\phi \cos\phi\ \partial h/\partial\phi +$$
$$+ \cos^2\lambda\ \partial^2 h/\partial\lambda^2 / \cos^2\phi - \cos\lambda \sin\lambda\ \partial h/\partial\lambda / \cos^2\phi -$$

$$- \cos\lambda \sin\lambda \sin\phi\ \partial^2 h/\partial\phi\partial\lambda / \cos\phi - \cos^2\lambda \sin\phi\ \partial h/\partial\phi / \cos\phi$$
$$- \sin\lambda \sin\phi \cos\lambda\ \partial^2 h/\partial\lambda\partial\phi / \cos\phi - \sin\lambda \sin^2\phi \cos\lambda\ \partial h/\partial\lambda / \cos^2\phi +$$
$$+ \sin^2\lambda \sin^2\phi\ \partial^2 h/\partial\phi^2 + \sin^2\lambda \sin\phi \cos\phi\ \partial h/\partial\phi + \cos^2\phi\ \partial^2 h/\partial\phi^2 - \cos\phi \sin\phi\ \partial h/\partial\phi\} / R^2 =$$

$$= \{\partial^2 h/\partial\lambda^2 / \cos^2\phi - \sin\phi\ \partial h/\partial\phi / \cos\phi + \sin^2\phi\ \partial^2 h/\partial\phi^2 + \sin\phi \cos\phi\ \partial h/\partial\phi -$$
$$+ \cos^2\phi\ \partial^2 h/\partial\phi^2 - \cos\phi \sin\phi\ \partial h/\partial\phi\} / R^2 =$$

$$= \{\partial^2 h/\partial\lambda^2 / \cos^2\phi - \sin\phi\ \partial h/\partial\phi / \cos\phi + \partial^2 h/\partial\phi^2\} / R^2 = X$$

$$\partial\lambda/\partial\mu = - rp / (pp+qq) = - \cos\lambda \tan\varphi$$
$$\partial\lambda/\partial\nu = - qr / (pp+qq) = - \sin\lambda \tan\varphi$$
$$\partial\varphi/\partial\mu = q / \sqrt{(pp+qq)} = \sin\lambda$$
$$\partial\varphi/\partial\nu = - p / \sqrt{(pp+qq)} = - \cos\lambda$$

$$\nabla^2 h = [\partial^2 h/\partial\mu^2 + \partial^2 h/\partial\nu^2 + \partial^2 h/\partial\lambda^2] / R^2 =$$

$$= [\partial(\partial\lambda/\partial\mu\ \partial h/\partial\lambda + \partial\phi/\partial\mu\ \partial h/\partial\phi)/\partial\mu + \partial(\partial\lambda/\partial\nu\ \partial h/\partial\lambda + \partial\phi/\partial\nu\ \partial h/\partial\phi)/\partial\nu + \partial^2 h/\partial\lambda^2] / R^2 =$$

$$= [\partial\lambda/\partial\mu\ \partial(\partial\lambda/\partial\mu\ \partial h/\partial\lambda + \partial\phi/\partial\mu\ \partial h/\partial\phi)/\partial\lambda + \partial\phi/\partial\mu\ \partial(\partial\lambda/\partial\mu\ \partial h/\partial\lambda + \partial\phi/\partial\mu\ \partial h/\partial\phi)/\partial\phi +$$
$$\partial\lambda/\partial\nu\ \partial(\partial\lambda/\partial\nu\ \partial h/\partial\lambda + \partial\phi/\partial\nu\ \partial h/\partial\phi)/\partial\lambda + \partial\phi/\partial\nu\ \partial(\partial\lambda/\partial\nu\ \partial h/\partial\lambda + \partial\phi/\partial\nu\ \partial h/\partial\phi)/\partial\phi +$$
$$+ \partial^2 h/\partial\lambda^2] / R^2 =$$

$$= [- \cos\lambda \tan\phi\ \partial(- \cos\lambda \tan\phi\ \partial h/\partial\lambda + \sin\lambda\ \partial h/\partial\phi)/\partial\lambda +$$
$$+ \sin\lambda\ \partial(- \cos\lambda \tan\phi\ \partial h/\partial\lambda + \sin\lambda\ \partial h/\partial\phi)/\partial\phi +$$
$$- \sin\lambda \tan\phi\ \partial(- \sin\lambda \tan\phi\ \partial h/\partial\lambda - \cos\lambda\ \partial h/\partial\phi)/\partial\lambda +$$
$$- \cos\lambda\ \partial(- \sin\lambda \tan\phi\ \partial h/\partial\lambda - \cos\lambda\ \partial h/\partial\phi)/\partial\phi + \partial^2 h/\partial\lambda^2] / R^2 =$$

$$= [- \sin\lambda \cos\lambda \tan^2\phi\ \partial h/\partial\lambda + \cos^2\lambda \tan^2\phi\ \partial^2 h/\partial\lambda^2 -$$
$$- \cos^2\lambda \tan\phi\ \partial h/\partial\phi - \sin\lambda \cos\lambda \tan\phi\ \partial(\partial h/\partial\phi)/\partial\lambda -$$
$$- \sin\lambda \cos\lambda\ (1 - \tan^2\phi)\ \partial h/\partial\lambda - \sin\lambda \cos\lambda \tan\phi\ \partial(\partial h/\partial\lambda)/\partial\phi +$$
$$+ \sin\lambda \cos\lambda\ \partial h/\partial\phi + \sin^2\lambda\ \partial^2 h/\partial\phi^2 +$$
$$+ \sin\lambda \cos\lambda \tan^2\phi\ \partial h/\partial\lambda + \sin^2\lambda \tan^2\phi\ \partial^2 h/\partial\lambda^2 -$$
$$- \sin^2\lambda \tan\phi\ \partial h/\partial\phi + \sin\lambda \cos\lambda \tan\phi\ \partial(\partial h/\partial\phi)/\partial\lambda +$$
$$+ \sin\lambda \cos\lambda\ (1 - \tan^2\phi)\ \partial h/\partial\lambda + \sin\lambda \cos\phi \tan\phi\ \partial(\partial h/\partial\lambda)/\partial\phi -$$
$$- \sin\lambda \cos\lambda\ \partial h/\partial\phi + \cos^2\lambda\ \partial^2 h/\partial\phi^2 + \partial^2 h/\partial\lambda^2] / R^2 =$$

$$= [\cos^2\lambda \tan^2\phi\, \partial^2h/\partial\lambda^2 - \cos^2\lambda \tan\phi\, \partial h/\partial\phi + \sin^2\lambda\, \partial^2h/\partial\phi^2 +$$
$$+ \sin^2\lambda \tan^2\phi\, \partial^2h/\partial\lambda^2 - \sin^2\lambda \tan\phi\, \partial h/\partial\phi + \cos^2\lambda\, \partial^2h/\partial\phi^2 + \partial^2h/\partial\lambda^2] / R^2 =$$

$$= [\tan^2\phi\, \partial^2h/\partial\lambda^2 - \tan\phi\, \partial h/\partial\phi + \partial^2h/\partial\phi^2 + \partial^2h/\partial\lambda^2] / R^2 =$$

$$= [\partial^2h/\partial\lambda^2 / \cos^2\phi - \sin\phi\, \partial h/\partial\phi / \cos\phi + \partial^2h/\partial\phi^2] / R^2 = X$$