Equivalence between new and old forms for ∇h

$\mathbf{W} = \mathbf{Z}\times\mathbf{P} / |\mathbf{Z}\times\mathbf{P}| = (-q, p, 0) / \sqrt{(pp+qq)} = (-\sin\lambda, \cos\lambda, 0)$
$\mathbf{N} = \mathbf{P}\times\mathbf{W} = (-pr, -qr, pp+qq) / \sqrt{(pp+qq)} = (-\cos\lambda \sin\phi, -\sin\lambda \sin\phi, \cos\phi)$

$\partial\lambda/\partial\delta = \mathbf{W}\bullet\mathbf{P}\times\mathbf{U} / \cos\phi = -\sin\lambda / \cos\phi \cos\delta$
$\partial\lambda/\partial\varepsilon = \mathbf{W}\bullet\mathbf{P}\times\mathbf{V} / \cos\phi = \cos\lambda / \cos\phi \cos\varepsilon$
$\partial\phi/\partial\delta = \mathbf{N}\bullet\mathbf{P}\times\mathbf{U} = \mathbf{W}\bullet\mathbf{U} = -\cos\lambda \sin\phi / \cos\delta$
$\partial\phi/\partial\varepsilon = \mathbf{N}\bullet\mathbf{P}\times\mathbf{V} = \mathbf{W}\bullet\mathbf{V} = -\sin\lambda \sin\phi / \cos\varepsilon$

$\nabla h = (\cos\delta\, \partial h/\partial\delta, \cos\varepsilon\, \partial h/\partial\varepsilon, \cos\phi\, \partial h/\partial\phi) / R =$

$= [\cos\delta\, (\partial\lambda/\partial\delta\, \partial h/\partial\lambda + \partial\phi/\partial\delta\, \partial h/\partial\phi), \cos\varepsilon\, (\partial\lambda/\partial\varepsilon\, \partial h/\partial\lambda + \partial\phi/\partial\varepsilon\, \partial h/\partial\phi), \cos\phi\, \partial h/\partial\phi] / R =$

$= [-\sin\lambda\, \partial h/\partial\lambda / \cos\phi - \cos\lambda \sin\phi\, \partial h/\partial\phi, \cos\lambda\, \partial h/\partial\lambda / \cos\phi - \sin\lambda \sin\phi\, \partial h/\partial\phi, \cos\phi\, \partial h/\partial\phi] / R$

$= [\mathbf{W}\, \partial h/\partial\lambda / \cos\phi + \mathbf{N}\, \partial h/\partial\phi] / R = (\mathbf{W}\, \partial h/\partial\lambda + \mathbf{N} \cos\phi\, \partial h/\partial\phi) / R \cos\phi$

$\partial\lambda/\partial\mu = -pr / (pp+qq) = -\cos\lambda \tan\phi$
$\partial\lambda/\partial\nu = -qr / (pp+qq) = -\sin\lambda \tan\phi$
$\partial\varphi/\partial\mu = q / \sqrt{(pp+qq)} = \sin\lambda$
$\partial\varphi/\partial\nu = -p / \sqrt{(pp+qq)} = -\cos\lambda$

$\nabla h = (r\, \partial h/\partial\nu - q\, \partial h/\partial\lambda, p\, \partial h/\partial\lambda - r\, \partial h/\partial\mu, q\, \partial h/\partial\mu - p\, \partial h/\partial\nu) / R =$

$= [r\, (\partial\lambda/\partial\nu\, \partial h/\partial\lambda + \partial\phi/\partial\nu\, \partial h/\partial\phi) - q\, \partial h/\partial\lambda, p\, \partial h/\partial\lambda - r\, (\partial\lambda/\partial\mu\, \partial h/\partial\lambda + \partial\phi/\partial\mu\, \partial h/\partial\phi),$
$q\, (\partial\lambda/\partial\mu\, \partial h/\partial\lambda + \partial\phi/\partial\mu\, \partial h/\partial\phi) - p\, (\partial\lambda/\partial\nu\, \partial h/\partial\lambda + \partial\phi/\partial\nu\, \partial h/\partial\phi)] / R =$

$= [-rqr\, \partial h/\partial\lambda / \cos^2\phi - rp\, \partial h/\partial\phi / \cos\phi - q\, \partial h/\partial\lambda,$
$p\, \partial h/\partial\lambda + rpr\, \partial h/\partial\lambda / \cos^2\phi - rp\, \partial h/\partial\phi / \cos\phi,$
$-qpr\, \partial h/\partial\lambda / \cos 2\phi + qq\, \partial h/\partial\phi / \cos\phi + pqr\, \partial h/\partial\lambda / \cos 2\phi + pp\, \partial h/\partial\phi / \cos\phi] / R =$

$= [-q\, \partial h/\partial\lambda / \cos^2\phi - rp\, \partial h/\partial\phi / \cos\phi, p\, \partial h/\partial\lambda / \cos^2\phi - rp\, \partial h/\partial\phi / \cos\phi, \cos\phi\, \partial h/\partial\phi] / R =$

$= [\mathbf{W}\, \partial h/\partial\lambda / \cos\phi + \mathbf{N}\, \partial h/\partial\phi] / R = (\mathbf{W}\, \partial h/\partial\lambda + \mathbf{N} \cos\phi\, \partial h/\partial\phi) / R \cos\phi$

$\mathbf{P}\times\nabla h = \mathbf{P}\times(r\,\partial h/\partial\nu - q\,\partial h/\partial\lambda,\ p\,\partial h/\partial\lambda - r\,\partial h/\partial\mu,\ q\,\partial h/\partial\mu - p\,\partial h/\partial\nu) / R$

The third component of $\mathbf{P}\times\nabla h$ is:

$[p\,(p\,\partial h/\partial\lambda - r\,\partial h/\partial\mu) - q\,(r\,\partial h/\partial\nu - q\,\partial h/\partial\lambda)] / R =$

$= [(pp + qq)\,\partial h/\partial\lambda - pr\,(\partial\lambda/\partial\mu\,\partial h/\partial\lambda + \partial\phi/\partial\mu\,\partial h/\partial\phi) - qr\,(\partial\lambda/\partial\nu\,\partial h/\partial\lambda + \partial\phi/\partial\nu\,\partial h/\partial\phi)] / R =$

$= \{(pp + qq)\,\partial h/\partial\lambda - pr\,[- pr\,\partial h/\partial\lambda / (pp+qq) + q\,\partial h/\partial\phi / \sqrt{(pp+qq)}] -$
$- qr\,[- qr\,\partial h/\partial\lambda / (pp+qq) - p\,\partial h/\partial\phi / \sqrt{(pp+qq)}]\} / R =$

$= \{(pp + qq)\,\partial h/\partial\lambda + prpr\,\partial h/\partial\lambda / (pp+qq) + qrqr\,\partial h/\partial\lambda / (pp+qq)\} / R =$

$= \{(pp + qq)\,\partial h/\partial\lambda + rr\,\partial h/\partial\lambda\} / R = \partial h/\partial\lambda / R$

Thus: $\mathbf{P}\times\nabla h = (\partial h/\partial\mu,\ \partial h/\partial\nu,\ \partial h/\partial\lambda) / R$