Equivalence between new and old forms for vertical component of $\nabla\times\mathbf{D}$

$\mathbf{W} = \mathbf{Z}\times\mathbf{P} / |\mathbf{Z}\times\mathbf{P}| = (-q, p, 0) / \sqrt{(pp+qq)}$
$\mathbf{N} = \mathbf{P}\times\mathbf{W} = (-pr, -qr, pp+qq) / \sqrt{(pp+qq)}$

$\mathbf{D}\bullet\mathbf{P} = 0 = dp + eq + fr = d \cos\lambda \cos\phi + e \sin\lambda \cos\phi + f \sin\phi$
$\mathbf{D}\bullet\mathbf{W} = (ep - qd) / \cos\phi$
$\mathbf{D}\bullet\mathbf{N} = (- dpr - eqr + fpp + fqq) / \cos\phi = (frr + fpp + fqq) / \cos\phi = f / \cos\phi$

$\partial\lambda/\partial\delta = \mathbf{W}\bullet\mathbf{P}\times\mathbf{U} / \cos\phi = - \sin\lambda / \cos\phi \cos\delta$
$\partial\lambda/\partial\varepsilon = \mathbf{W}\bullet\mathbf{P}\times\mathbf{V} / \cos\phi = \cos\lambda / \cos\phi \cos\varepsilon$
$\partial\phi/\partial\delta = \mathbf{N}\bullet\mathbf{P}\times\mathbf{U} = \mathbf{W}\bullet\mathbf{U} = - \cos\lambda \sin\phi / \cos\delta$
$\partial\phi/\partial\varepsilon = \mathbf{N}\bullet\mathbf{P}\times\mathbf{V} = \mathbf{W}\bullet\mathbf{V} = - \sin\lambda \sin\phi / \cos\varepsilon$

$$\mathbf{P}\bullet\nabla\times\mathbf{D} = [p (\cos\varepsilon\, \partial f/\partial\varepsilon - \cos\phi\, \partial e/\partial\phi) + q (\cos\phi\, \partial d/\partial\phi - \cos\delta\, \partial f/\partial\delta) + r (\cos\delta\, \partial e/\partial\delta - \cos\varepsilon\, \partial d/\partial\varepsilon)] / R =$$

$$= \{\cos\lambda \cos\phi [\cos\varepsilon (\partial\lambda/\partial\varepsilon\, \partial f/\partial\lambda + \partial\phi/\partial\varepsilon\, \partial f/\partial\phi) - \cos\phi\, \partial e/\partial\phi] + \sin\lambda \cos\phi [\cos\phi\, \partial d/\partial\phi - \cos\delta (\partial\lambda/\partial\delta\, \partial f/\partial\lambda + \partial\phi/\partial\delta\, \partial f/\partial\phi)] + \sin\phi [\cos\delta (\partial\lambda/\partial\delta\, \partial e/\partial\lambda + \partial\phi/\partial\delta\, \partial e/\partial\phi) - \cos\varepsilon (\partial\lambda/\partial\varepsilon\, \partial d/\partial\lambda + \partial\phi/\partial\varepsilon\, \partial d/\partial\phi)]\} / R =$$

$$= \{\cos\lambda \cos\phi [\cos\lambda\, \partial f/\partial\lambda / \cos\phi - \sin\lambda \sin\phi\, \partial f/\partial\phi - \cos\phi\, \partial e/\partial\phi] + \sin\lambda \cos\phi [\cos\phi\, \partial d/\partial\phi + \sin\lambda\, \partial f/\partial\lambda / \cos\phi - \cos\lambda \sin\phi\, \partial f/\partial\phi] + \sin\phi [- \sin\lambda\, \partial e/\partial\lambda / \cos\phi - \cos\lambda \sin\phi\, \partial e/\partial\phi - \cos\lambda\, \partial d/\partial\lambda / \cos\phi + \sin\lambda \sin\phi\, \partial d/\partial\phi]\} / R =$$

$$= \{\cos2\lambda\, \partial f/\partial\lambda - \cos\lambda \cos2\phi\, \partial e/\partial\phi + \sin\lambda \cos2\phi\, \partial d/\partial\phi + \sin2\lambda\, \partial f/\partial\lambda - \sin\lambda \tan\phi\, \partial e/\partial\lambda - \cos\lambda \sin2\phi\, \partial e/\partial\phi - \cos\lambda \tan\phi\, \partial d/\partial\lambda + \sin\lambda \sin2\phi\, \partial d/\partial\phi\} / R =$$

$$= \{\partial f/\partial\lambda - \cos\lambda\, \partial e/\partial\phi + \sin\lambda\, \partial d/\partial\phi - \sin\lambda \tan\phi\, \partial e/\partial\lambda - \cos\lambda \tan\phi\, \partial d/\partial\lambda\} / R =$$

$$= \{\partial f/\partial\lambda - \cos\lambda [\partial(e \cos\phi)/\partial\phi + e \sin\phi] / \cos\phi + \sin\lambda [\partial(d \cos\phi)/\partial\phi + d \sin\phi] / \cos\phi - \tan\phi [\partial(e \sin\lambda)/\partial\lambda - e \cos\lambda] - \tan\phi [\partial(d \cos\lambda)/\partial\lambda + d \sin\lambda]\} / R =$$

$$= \{\partial f/\partial\lambda - \cos\lambda\, \partial(e \cos\phi)/\partial\phi / \cos\phi + \sin\lambda\, \partial(d \cos\phi)/\partial\phi / \cos\phi - \tan\phi\, \partial(e \sin\lambda)/\partial\lambda - \tan\phi\, \partial(d \cos\lambda)/\partial\lambda\} / R =$$

$= \{\partial f/\partial\lambda - \partial(ep - dq)/\partial\phi / \cos\phi - \tan\phi\ \partial(eq + dp)/\partial\lambda / \cos\phi\} / R =$

$= \{\cos\phi\ \partial f/\partial\lambda - \partial(\mathbf{D}\bullet\mathbf{W}\cos\phi)/\partial\phi + \tan\phi\ \partial(fr)/\partial\lambda\} / R\cos\phi =$

$= \{\cos 2\phi\ \partial(f / \cos\phi)/\partial\lambda - \partial(\mathbf{D}\bullet\mathbf{W}\cos\phi)/\partial\phi + \sin 2\phi\ \partial(f / \cos\phi)/\partial\lambda\} / R\cos\phi =$

$= [\partial(\mathbf{D}\bullet\mathbf{N})/\partial\lambda - \partial(\mathbf{D}\bullet\mathbf{W}\cos\phi)/\partial\phi] / R\cos\phi =$

$\partial\lambda/\partial\mu = -rp / (pp+qq) = -\cos\lambda\tan\varphi$
$\partial\lambda/\partial\nu = -qr / (pp+qq) = -\sin\lambda\tan\varphi$
$\partial\varphi/\partial\mu = q / \sqrt{(pp+qq)} = \sin\lambda$
$\partial\varphi/\partial\nu = -p / \sqrt{(pp+qq)} = -\cos\lambda$

$\mathbf{P}\bullet\nabla\times\mathbf{D} = (\partial d/\partial\mu, \partial e/\partial\nu, \partial f/\partial\lambda) / R =$

$= (\partial\lambda/\partial\mu\ \partial d/\partial\lambda + \partial\phi/\partial\mu\ \partial d/\partial\phi + \partial\lambda/\partial\nu\ \partial e/\partial\lambda + \partial\phi/\partial\nu\ \partial e/\partial\phi + \partial f/\partial\lambda) / R =$

$= (-\cos\lambda\tan\phi\ \partial d/\partial\lambda + \sin\lambda\ \partial d/\partial\phi - \sin\lambda\tan\phi\ \partial e/\partial\lambda - \cos\lambda\ \partial e/\partial\phi + \partial f/\partial\lambda) / R =$

$= (-\cos\lambda\sin\phi\ \partial d/\partial\lambda - \sin\lambda\sin\phi\ \partial e/\partial\lambda + \cos\phi\ \partial f/\partial\lambda +$
$+ \sin\lambda\cos\phi\ \partial d/\partial\phi - \cos\lambda\cos\phi\ \partial e/\partial\phi) / R\cos\phi =$

$= \{-\sin\phi\ [\partial(d\cos\lambda)/\partial\lambda + d\sin\lambda] - \sin\phi\ [\partial(e\sin\lambda)/\partial\lambda - e\cos\lambda] + \cos\phi\ \partial f/\partial\lambda +$
$+ \sin\lambda\ [\partial(d\cos\phi)/\partial\phi + d\sin\phi] - \cos\lambda\ [\partial(e\cos\phi)/\partial\phi + e\sin\phi]\} / R\cos\phi =$

$= \{-\sin\phi\ \partial(d\cos\lambda)/\partial\lambda - \sin\phi\ \partial(e\sin\lambda)/\partial\lambda + \cos\phi\ \partial f/\partial\lambda +$
$+ \sin\lambda\ \partial(d\cos\phi)/\partial\phi - \cos\lambda\ \partial(e\cos\phi)/\partial\phi\} / R\cos\phi =$

$= \{-\sin\phi\ \partial(d\cos\lambda\cos\phi + e\sin\lambda\cos\phi)/\partial\lambda / \cos\phi + \cos\phi\ \partial f/\partial\lambda +$
$- \partial(e\cos\lambda\cos\phi - d\sin\lambda\cos\phi)/\partial\phi\} / R\cos\phi =$

$= \{\sin\phi\ \partial(f\sin\phi)/\partial\lambda / \cos\phi + \cos\phi\ \partial f/\partial\lambda - \partial(\mathbf{D}\bullet\mathbf{W}\cos\phi)/\partial\phi\} / R\cos\phi =$

$= \{\partial(f / \cos\phi)/\partial\lambda - \partial(\mathbf{D}\bullet\mathbf{W}\cos\phi)/\partial\phi\} / R\cos\phi = [\partial(\mathbf{D}\bullet\mathbf{N})/\partial\lambda - \partial(\mathbf{D}\bullet\mathbf{W}\cos\phi)/\partial\phi] / R\cos\phi$

Proof that if D is horizontal, then $(\nabla\times\mathbf{D})\times\mathbf{P} = \mathbf{D} / R$

$\partial\lambda/\partial\delta = \mathbf{W}\bullet\mathbf{P}\times\mathbf{U} / \cos\phi = -q / (pp+qq)\sqrt{(qq+rr)} = -\sin\lambda / \cos\phi\cos\delta$
$\partial\lambda/\partial\varepsilon = \mathbf{W}\bullet\mathbf{P}\times\mathbf{V} / \cos\phi = p / (pp+qq)\sqrt{(rr+pp)} = \cos\lambda / \cos\phi\cos\varepsilon$
$\partial\phi/\partial\delta = \mathbf{N}\bullet\mathbf{P}\times\mathbf{U} = \mathbf{W}\bullet\mathbf{U} = -pr / \sqrt{(pp+qq)}\sqrt{(qq+rr)} = -\cos\lambda\sin\phi / \cos\delta$
$\partial\phi/\partial\varepsilon = \mathbf{N}\bullet\mathbf{P}\times\mathbf{V} = \mathbf{W}\bullet\mathbf{V} = -qr / \sqrt{(pp+qq)}\sqrt{(qq+rr)} = -\sin\lambda\sin\phi / \cos\delta$

$\mathbf{Z}\bullet(\nabla\times\mathbf{D})\times\mathbf{P} = [(\cos\varepsilon\ \partial f/\partial\varepsilon - \cos\phi\ \partial e/\partial\phi)\ q - (\cos\phi\ \partial d/\partial\phi - \cos\delta\ \partial f/\partial\delta)\ p] / R =$

$= [p\cos\delta\ (\partial f/\partial\lambda + \partial\phi/\partial\delta\ \partial f/\partial\phi) + q\cos\varepsilon\ (\partial\lambda/\partial\varepsilon\ \partial f/\partial\lambda + \partial\phi/\partial\varepsilon\ \partial f/\partial\phi) -$
$\quad - p\cos\phi\ \partial d/\partial\phi - q\cos\phi\ \partial e/\partial\phi)] / R =$

$= \{p\ (-\sin\lambda\ \partial f/\partial\lambda / \cos\phi - \cos\lambda\sin\phi\ \partial f/\partial\phi) + q\ (\cos\lambda\ \partial f/\partial\lambda / \cos\phi - \sin\lambda\sin\phi\ \partial f/\partial\phi) -$
$\quad - p\ [\partial(d\cos\phi)/\partial\phi + d\sin\phi] - q\ [\partial(e\cos\phi)/\partial\phi + e\sin\phi]\} / R =$

$= \{p\ (-\cos\lambda\sin\phi\ \partial f/\partial\phi) + q\ (-\sin\lambda\sin\phi\ \partial f/\partial\phi) -$
$\quad - \cos\phi\ \partial(d\cos\lambda\cos\phi)/\partial\phi - \cos\phi\ \partial(e\sin\lambda\cos\phi)/\partial\phi - pdr - qer\} / R =$

$= \{-\cos^2\lambda\cos\phi\sin\phi\ \partial f/\partial\phi - \sin^2\lambda\cos\phi\sin\phi\ \partial f/\partial\phi - \cos\phi\ \partial(-f\sin\phi)/\partial\phi + rfr\} / R =$

$= \{-\cos\phi\sin\phi)\ \partial f/\partial\phi + \cos\phi\sin\phi\ \partial f/\partial\phi + f\cos^2\phi + f\sin^2\phi\} / R = f / R$

Similarly, $\mathbf{X}\bullet(\nabla\times\mathbf{D})\times\mathbf{P} = d / R$, $\mathbf{Y}\bullet(\nabla\times\mathbf{D})\times\mathbf{P} = e$, and
$(\nabla\times\mathbf{D})\times\mathbf{P} = \mathbf{D} / R$