Advection at a randomly positioned spherical edge

$\mathbf{I} = (i, j, k) =$ coordinates of "north pole"
$\xi =$ longitudinal angular rotation variable around $\mathbf{I}$ axis
$\eta =$ latitudinal angular rotation variable from $-\mathbf{I}$ to $\mathbf{I}$
$\sin\eta = \mathbf{I}\bullet\mathbf{P} = ip + jq + kr$
$\cos\eta = |\mathbf{I}\times\mathbf{P}| = \sqrt{[(jr-kq)^2 + (kp-ir)^2 + (iq-jp)^2]}$
$\mathbf{F} = \mathbf{I}\times\mathbf{P} / |\mathbf{I}\times\mathbf{P}| = (jr-kq, kp-ir, iq-jp) / \cos\eta =$ unit vector perpendicular to edge
$\mathbf{E} = \mathbf{P}\times\mathbf{F} = (iqq+irr-jpq-krp, jrr+jpp-kqr-ipq, kpp+kqq-irp-jqr) / \cos\eta =$
$= (i - p\sin\eta, j - q\sin\eta, k - r\sin\eta) / \cos\eta$
$f = \mathbf{F}\bullet\mathbf{S} = w\,\mathbf{F}\bullet\mathbf{W} + n\,\mathbf{F}\bullet\mathbf{N} =$ velocity component perpendicular to edge
$e = \mathbf{E}\bullet\mathbf{S} = w\,\mathbf{E}\bullet\mathbf{W} + n\,\mathbf{E}\bullet\mathbf{N} =$ velocity component parallel to edge

$\mathbf{F}\bullet\mathbf{W} = (kpp + kqq - ipr - jqr) / \cos\eta\cos\phi = (k - r\sin\eta) / \cos\eta\cos\phi =$
$= (k\cos\phi - i\cos\lambda\sin\phi - j\sin\lambda\sin\phi) / \cos\eta$
$\mathbf{E}\bullet\mathbf{W} = (jp - iq) / \cos\eta\cos\phi = (j\cos\lambda - i\sin\lambda) / \cos\eta$
$\mathbf{F}\bullet\mathbf{N} = (iq - jp) / \cos\eta\cos\phi = (i\sin\lambda - j\cos\lambda) / \cos\eta = -\mathbf{E}\bullet\mathbf{W}$
$\mathbf{E}\bullet\mathbf{N} = (kpp + kqq - ipr - jqr) / \cos\eta\cos\phi = (k - r\sin\eta) / \cos\eta\cos\phi = \mathbf{F}\bullet\mathbf{W}$
$(\mathbf{F}\bullet\mathbf{W})^2 + (\mathbf{E}\bullet\mathbf{W})^2 = (\mathbf{F}\bullet\mathbf{N})^2 + (\mathbf{E}\bullet\mathbf{N})^2 = (\mathbf{F}\bullet\mathbf{W})^2 + (\mathbf{F}\bullet\mathbf{N})^2 = (\mathbf{E}\bullet\mathbf{W})^2 + (\mathbf{E}\bullet\mathbf{N})^2 = 1$

$\partial\lambda/\partial\xi = \mathbf{W}\bullet\mathbf{F}\cos\eta / \cos\phi = (kpp + kqq - ipr - jqr) / \cos^2\phi$
$\partial\lambda/\partial\eta = \mathbf{W}\bullet\mathbf{E} / \cos\phi = (jp - iq) / \cos\eta\cos^2\phi$
$\partial\phi/\partial\xi = \mathbf{N}\bullet\mathbf{F}\cos\eta = (iq - jp) / \cos\phi$
$\partial\phi/\partial\eta = \mathbf{N}\bullet\mathbf{E} = (kpp + kqq - ipr - jqr) / \cos\eta\cos\phi$

$\partial\mathbf{A}/\partial t + (f\,\partial\mathbf{A}/\partial\xi + e\cos\eta\,\partial\mathbf{A}/\partial\eta) / R\cos\eta =$

$\partial\mathbf{A}/\partial t + [f\,(\partial\lambda/\partial\xi\,\partial\mathbf{A}/\partial\lambda + \partial\phi/\partial\xi\,\partial\mathbf{A}/\partial\phi) + e\cos\eta\,(\partial\lambda/\partial\eta\,\partial\mathbf{A}/\partial\lambda + \partial\phi/\partial\eta\,\partial\mathbf{A}/\partial\phi)] / R\cos\eta =$

$\partial\mathbf{A}/\partial t + [(w\,\mathbf{F}\bullet\mathbf{W} + n\,\mathbf{F}\bullet\mathbf{N})\,(\mathbf{W}\bullet\mathbf{F}\cos\eta\,\partial\mathbf{A}/\partial\lambda / \cos\phi + \mathbf{N}\bullet\mathbf{F}\cos\eta\,\partial\mathbf{A}/\partial\phi) +$
$+ (w\,\mathbf{E}\bullet\mathbf{W} + n\,\mathbf{E}\bullet\mathbf{N})\cos\eta\,(\mathbf{W}\bullet\mathbf{E}\,\partial\mathbf{A}/\partial\lambda / \cos\phi + \mathbf{N}\bullet\mathbf{E}\,\partial\mathbf{A}/\partial\phi)] / R\cos\eta =$

$\partial\mathbf{A}/\partial t + \{w\,\partial\mathbf{A}/\partial\lambda\,[(\mathbf{F}\bullet\mathbf{W})^2 + (\mathbf{W}\bullet\mathbf{E})^2] / \cos\phi + w\,\partial\mathbf{A}/\partial\phi\,[\mathbf{W}\bullet\mathbf{F}\,\mathbf{N}\bullet\mathbf{F} + \mathbf{W}\bullet\mathbf{E}\,\mathbf{N}\bullet\mathbf{E}] +$
$+ n\,\partial\mathbf{A}/\partial\lambda\,[\mathbf{N}\bullet\mathbf{F}\,\mathbf{W}\bullet\mathbf{F} + \mathbf{N}\bullet\mathbf{E}\,\mathbf{W}\bullet\mathbf{E}] / \cos\phi + n\,\partial\mathbf{A}/\partial\phi\,[(\mathbf{N}\bullet\mathbf{F})^2 + (\mathbf{N}\bullet\mathbf{E})^2] / R =$

$\partial\mathbf{A}/\partial t + \{w\,\partial\mathbf{A}/\partial\lambda / \cos\phi + n\,\partial\mathbf{A}/\partial\phi\} / R = \partial\mathbf{A}/\partial t + (w\,\partial\mathbf{A}/\partial\lambda + n\cos\phi\,\partial\mathbf{A}/\partial\phi) / R\cos\phi$